MATHEMATICS

Paper 9709/01
Paper 1

General comments

Most candidates found the paper accessible and there were a large number of very good scripts. There were only a few scripts from candidates who should not have been entered for the paper. Whilst candidates coped reasonably with most of the questions, the need to use algebra in **Question 4** presented problems as did the need to use exact values for sine and cosine in **Question 7**. Although most candidates can now cope with finding the inverse of a quadratic expression, a small minority of candidates realised the need to take the negative root in **Question 11**.

Comments on specific questions

Question 1

This question proved to be a good starting question. The majority of candidates eliminated y correctly from the two equations and most realised that the discriminant ($b^2 - 4ac$) of the resulting equation must be negative. Unfortunately the subsequent algebra often proved too complex and only about a half of all solutions obtained the answer $k < -4$.

Answer: $k < -4$.

Question 2

A large proportion of candidates obtained a correct answer. The standard of integration and subsequent use of limits was very good. Common errors were to use an incorrect formula for the area under a curve, usually either $\int y^2 \mathrm{d}x$ or $\pi\int y \mathrm{d}x$, or to assume that $\sqrt{x}$ equalled either x^{-1} or $x^{-\frac{1}{2}}$.

Answer: $9\frac{1}{3}$.

Question 3

(i) This was nearly always correctly answered, though there were some candidates who failed to realise the meaning of 'ascending'. The use of binomial coefficients was very pleasing.

(ii) A majority of attempts realised that the coefficient of x^2 came from the sum of two terms and there were many completely correct solutions.

Answers: **(i)** $32 + 80u + 80u^2$; **(ii)** 160.

Question 4

(i) Candidates had no difficulty in expressing the 5th term as $a + 4d$ and the 15th term as $a + 14d$. Unfortunately many candidates failed to realise that if a, b and c are in geometric progression, then $ac = b^2$. Many candidates stated that $a + 4d = ar$ and that $a + 14d = ar^2$, and attempted to eliminate r. Errors in the subsequent algebra were common, especially in assuming that $(a + 4d)^2 = a^2 + 16d^2$.

(ii) Correct answers were rare, with many candidates being unable to untangle a lot o[illegible] algebra.

Answers: **(i)** $a + 4d$, $a + 14d$; **(iii)** 2.5.

Question 5

(i) This was usually very well answered, with candidates confidently using the identities $\frac{\sin x}{\cos x} = \tan x$ and $\sin^2 x + \cos^2 x = 1$.

(ii) Whilst the majority had no problems in solving the quadratic equation from part **(i)**, a small minority failed to link the two parts, and others surprisingly included an extra '8' in the quadratic equation. Most candidates solved the equation correctly and rejected solutions from $\cos^{-1}(-3)$. The majority also realised that there were two solutions; one acute, the other in the 4th quadrant.

Answers: **(ii)** 70.5º, 289.5º.

Question 6

The majority of candidates obtained full marks for the question. A surprising number however found the equations of *AB* and *AD* and finished with the coordinates of *A*. Whilst the majority realised the need to find the equation of the perpendicular, there were several solutions in which the relationship between perpendicular gradients was taken as $m_1m_2 = 1$, rather than −1. Most also realised that the gradients of *AB* and *DC* were the same and the solution of the simultaneous equations was generally accurate.

Answer: (6.2, 9.6).

Question 7

(i) Candidates need to realise, that even if a 'proof' can be visualised mentally, there is a need to show full working. Whilst most candidates realised that the area of the sector was $\frac{1}{2}r^2\theta$, there was considerable difficulty in finding the expression for the area of the triangle *OAX*. Many expressions were obviously 'fiddled', and there were a significant number of candidates who failed to realise the need to use trigonometry in the triangle *OAX*.

(ii) There were many solutions in which $\sin\left(\frac{\pi}{6}\right)$ and $\cos\left(\frac{\pi}{6}\right)$ were interchanged, and many others which failed to realise the need to use the exact surd form of $\cos\left(\frac{\pi}{6}\right)$. Decimal answers were common with many candidates still failing to realise that 'exact' effectively rules out any use of a calculator.

Answer: **(ii)** $18 - 6\sqrt{3} + 2\pi$.

Question 8

(i) This was well answered and it was pleasing that the majority of candidates realised that the function was composite and multiplied by the differential of the bracket $(2x - 3)$. A small minority attempted to expand and then differentiate $(2x-3)^3$, but only a few were able to do this accurately. Surprisingly, there were several instances when the '−6' appeared in the second differential.

(ii) Although most candidates realised that the differential was zero, there were a significant number who incorrectly set the second differential to zero. Knowledge of the use of the second differential to determine the nature of the stationary point was consistently good.

Answers: **(i)** $6(2x-3)^2 - 6$, $24(2x-3)$; **(ii)** Minimum at $x = 2$, Maximum at $x = 1$.

Question 9

(i) Although most realised the need to integrate, it was disappointing to see a large number of candidates using $y = mx + c$ with $m = \frac{dy}{dx} = 4 - x$. Some of these compounded the error by leaving m in terms of x, others substituted $m = 2$, thinking that the equation of the tangent was the same as the equation of the curve. Of those realising the need to integrate, a large number failed to include the constant of integration.

(ii) Candidates were more successful in this part. Most had no difficulty in proceeding from the gradient of the tangent to the gradient of the normal. The common error was to take the gradient of the tangent as 4, sometimes from the '$4x$' in the equation of the curve, and sometimes from the '4' in $4 - x$.

(iii) Most candidates realised the need to solve the equations of the curve and the normal simultaneously. Only a minority of attempts however were completely correct.

Answers: **(i)** $y = 4x - \frac{1}{2}x^2 + 3$; **(ii)** $2y + x = 20$; **(iii)** (7, 6.5).

Question 10

(i) Most candidates attempted this by direct reference to the diagram, working firstly from P to R, and then from P to Q, in each of the **i**, **j** and **k** directions.

(ii) The scalar product was very well done and most candidates obtained the 3 method marks available. A small minority made the mistake of assuming that angle QPR needed $\overrightarrow{QP}.\overrightarrow{PR}$ and found the obtuse instead of the acute angle.

(iii) Most candidates realised the need to find the length of each side of the triangle by Pythagoras, but many failed to realise the need to find vector $\overrightarrow{QR}$ first. Many assumed that the triangle was isosceles, or even in a few cases equilateral.

Answers: **(i)** 2**i** + 2**j** + 2**k**, −2**i** + 2**j** + 4**k**; **(ii)** 61.9°; **(iii)** 12.8 cm.

Question 11

(i) Only about a half of all attempts were correct. The successful candidates nearly always removed the '2' completely and worked with the remainder before reinstalling the '2'.

(ii) Most candidates realised that the range of a quadratic function could be written directly from the answer to part **(i)** i.e. $f(x) \geqslant$ 'c'.

(iii) The majority of candidates realised that the function had no inverse because it was not one-one.

(iv) This was poorly answered with only a small number of candidates realising that the answer could also be obtained directly from $a(x+b)^2 + c$, i.e. $A = -b$.

(v) Using the answer to part (**i**) to find an expression for $g^{-1}(x)$ was well done. Unfortunately only a few candidates realised that taking the square root requires '±', and that in this case simple testing requires the '−' rather than the '+' sign. Most realised that the range of g^{-1} is the same as the domain of g.

Answers: **(i)** $2(x-2)^2 + 3$; **(ii)** $f(x) \geqslant 3$; **(iii)** f is not one-one; **(iv)** 2; **(v)** $2 - \sqrt{\left(\frac{x-3}{2}\right)}$, $g^{-1}(x) \leqslant 2$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

There was a wide range of candidate performance. There were many scripts which scored highly; however there were also many candidates who produced work which merited only single figure marks out of 50.

Comments on specific questions

Question 1

Many candidates realised that they had to look at ln(2x + 1) and errors in using the appropriate constant of $\frac{1}{2}$ were common. The given answer encouraged some candidates to produce an apparently correct answer from incorrect work. A typical error was $\frac{1}{2}(\ln 9 - \ln 3) = \frac{1}{2}\left(\frac{\ln 9}{\ln 3}\right)$ following on to the given result of $\frac{1}{2}\ln 3$.

Question 2

(i) Most candidates were able to make a reasonable attempt at the iteration. However many candidates failed to gain marks by not giving either their workings to 4 decimal places as requested or their final answer to 2 decimal places as requested. Those candidates that made arithmetic errors in some of their iterations very often recovered and were able to gain credit for a correct result.

(ii) Completely correct solutions were in the minority and many candidates misunderstood what was being asked of them.

Answers: **(i)** 2.29; **(ii)** $\sqrt[3]{12}$.

Question 3

(i) Most candidates were able to deal with the idea of a modulus and were able to obtain the appropriate critical values, usually without having to resort to use of a quadratic equation. Those candidates that did make use of a quadratic equation sometimes made errors in the factorisation, thus obtaining 2 incorrect critical values which then affected the next part of the question. Most were able to find the correct range for the inequality.

(ii) Many candidates were able to link correctly this part of the question with the previous part and were able to use logarithms correctly to obtain either one or both of the critical values. Correct ranges were less common in this part, many candidates forgetting that they had to carry on and give their answer in the form of an inequality. Those candidates who obtained incorrect critical values in the first part of the question, were often able to use a correct method with their values and gain some credit.

Answers: **(i)** $4 < y < 6$; **(ii)** $1.26 < x < 1.63$.

Question 4

Completely correct solutions to this question were extremely few. Most candidates were able to diff correctly and equate their result to 0. Problems arose with the solution of the resulting equation. candidates did not know that $\sec x = \frac{1}{\cos x}$, others had problems with dealing with the square root and th trigonometric ratio involved in the solution of the equation. For those that did reach a correct result of $\cos x = \pm\frac{1}{\sqrt{2}}$ solutions outside the given range were often produced. Few candidates realised that the *y* coordinate also had to be found.

Answer: $(\frac{1}{4}\pi, \frac{1}{2}\pi - 1)$, $(-\frac{1}{4}\pi, -\frac{1}{2}\pi + 1)$

Question 5

(i) Most candidates were able to substitute $x = -2$ into the given polynomial and obtain the correct solution.

(ii) Solutions to this part of the question were usually done well by the majority of candidates who used a variety of methods to obtain a quadratic factor. Too many candidates did not read the question carefully and gave their final answer as a set of 3 linear factors. Others, while giving their solutions to the quadratic equation they had found, forgot to give the solution of $x = -2$.

Answers: **(i)** 3; **(ii)** -2, -1, $\frac{1}{3}$.

Question 6

(i) This was another example of candidates failing to read the question carefully and thus losing marks. While many were able to obtain R and α using correct methods, too many failed to give α to the required level of accuracy. The most common error apart from this was to use $\tan\alpha = \frac{8}{15}$.

(ii) Many candidates were able to produce correct solutions to this part of the question, with occasional lapses in accuracy and with spurious additional solutions sometimes making an appearance.

Answers: **(i)** 17, 61.9°; **(ii)** 117.4°, 186.5°.

Question 7

(i) Correct proofs of the identity were very rare. Many candidates were able to expand out the left hand side of the identity and apply usually only one of the appropriate double angle formulae. Those candidates that were able to use more than one of the double angle formulae invariably were unable to deal with the simplification of the algebra involved. Those candidates who chose to start with the right hand side of the identity, whilst being able to use the correct double angle formulae, were unable to deal with the algebraic manipulation to obtain the correct result.

(ii) Most candidates who attempted this part of the question realised that they needed to use the result from the first part and made attempts to integrate each term. This was rarely done correctly, with many errors occurring in the constants involved. Those candidates that did integrate correctly failed to realise the significance of the request for an exact answer and resorted to using their calculators for an answer in decimal form.

Answer: **(ii)** $\frac{1}{4}(5\pi - 2)$ or exact simplified equivalent.

Question 8

(i) Most candidates who attempted this question, realised that they had to use either the pro… or the quotient rule. Many errors were made both in the differentiation of the exponential ter… the resulting simplification. There were many correct solutions to this part, but in the solution of… derivative set to 0, many candidates incorrectly produced solutions from the exponential term se… to 0.

(ii) This part of the question was rarely done correctly. Many candidates appeared to misunderstand what was being asked of them. There were some attempts to obtain the equation of the tangent at the appropriate point, but many candidates resorted to use of rounded figures rather than exact ones, very often rounding to an unacceptable degree of accuracy. Many candidates failed to show the required result by not showing or explaining that this tangent went through the origin.

(iii) Many completely correct solutions to this part were seen, showing that many candidates had a good understanding of the topic. There were some candidates who insisted on using the origin as one of the points and other who made errors in the evaluation of the values required.

Answers: **(i)** 2; **(iii)** 0.95.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

The standard of work on this paper varied considerably and resulted in a broad spread of marks. Well prepared candidates appeared to have sufficient time to answer all questions and no question seemed to be of undue difficulty, though completely correct solutions to **Question 7** were rare. The questions or parts of questions that were done particularly well were **Question 5** (trigonometry), **Question 8** (complex numbers) and **Question 9** (partial fractions). Those that were found the most difficult were **Question 7** (differential equation) and **Question 10** (vector geometry). Overall the main weakness was the work involving calculus.

In general the presentation of work was good but there were still a few candidates who presented their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards answers or statements given in the question paper, for example as in **Question 3**, **Question 5(i)** and **Question 6(i)** and **(ii)**. Examiners penalise the omission of essential working in such cases.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Some candidates answered this well, but many made careless errors. Most candidates stated an indefinite integral of the from $a\ln(2x - 1)$. If the value of a was incorrect then it was usually 2 or 1. Poor handling of the brackets was common. For example, the logarithm was treated as $\ln(2x) - 1$ or $\ln(2x) - \ln 1$, or $2\ln(x - 1)$. At the end, some candidates who had reached a correct exact answer wasted time by evaluating the answer to some degree of accuracy; others went directly from $2k - 1 = e^2$ to a numerical answer and forfeited the mark reserved for the statement of the exact answer.

Answer: $\frac{1}{2}(e^2 + 1)$.

Question 2

This question was generally found to be straightforward by candidates. Most divided the quartic by the given factor, thereby producing the other quadratic factor as the quotient of the division. Occasionally there was a careless mistake in the division process, for example taking the product of $x^2 + x + 2$ and 2 to be $2x^2 + 2x + 2$.

Another method was to write the other quadratic factor as $x^2 + bx + c$, multiply by $x^2 + x + 2$ and compare the result with $x^4 + 3x^2 + a$. This was generally done well.

Answers: $a = 4$; $x^2 - x + 2$.

Question 3

The majority of candidates attempted to use integration by parts and the question was quite well a
Some who reached $x\ln x - \int x.\frac{1}{x}\,\mathrm{d}x$ thought that the remaining integral equalled the product of the integ
of x and $\frac{1}{x}$. The manipulation of logarithms needed to reach the given answer seemed to be beyond some candidates, while others failed to give sufficient working to earn the final mark.

Question 4

(i) Differentiation using the product or quotient was usually done well. Most candidates set the derivative equal to zero and tried to solve for x. It was quite common to see the statement $\sin x = \cos x$ being followed by $\tan x = 0$, and candidates finding spurious solutions to the equation $\mathrm{e}^{-x} = 0$.

(ii) Most attempts at determining the nature of the stationary point were based on the second derivative. Differentiation of the first derivative was marred by frequent errors of sign and of method. Some candidates with a correct second derivative, $-2\mathrm{e}^{-x}\cos x$, simply said that it was negative without making any reference to their value of x. Possibly they thought that such a function was negative for all values of x.

Answers: **(i)** $\frac{1}{4}\pi$ or 0.785 radians; **(ii)** Maximum.

Question 5

This was generally well answered. In part **(i)** most candidates derived the given answer and gave sufficient working to justify it. In part **(ii)** some mistakenly equated the given quadratic to 2 rather than zero. Those who worked with the given quadratic usually found the correct answer in the first quadrant, but the value in the second quadrant caused some problems. Premature approximation of the solutions to the quadratic in $\tan x$ was also a source of error.

Answers: **(ii)** 22.5°, 112.5°.

Question 6

(i) Relatively few candidates gained full marks here. Most of the sketches of the line $y = 2 - x$ were adequate but many sketches of $y = \ln x$ were not. Some of the latter had the wrong curvature, some only showed the part of the curve for $x \geqslant 1$, some were straight lines and some resembled $y = \mathrm{e}^x$. Even when both curves were correctly sketched and provided sufficient evidence for a conclusion to be drawn, many candidates failed to complete the exercise by explicitly stating that the existence of just one point of intersection implied that the equation had only one root.

(ii) This part was also poorly answered. Some candidates seemed to believe that a statement involving ‘positive’ and ‘negative’ was sufficient, without any reference to there being a change of sign, or even the function under consideration. However others did make clear the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

(iii) This was generally well answered.

(iv) Most candidates gave the result of each iteration to 4 decimal places as required, though some failed to give the final answer to 2 decimal places.

Answer: **(iv)** 1.56.

Question 7

This question differentiated well. Part **(i)** was reasonably well answered. However a few candida unable even to separate variables correctly and their answers were consequently worthless. For thos separated correctly, a common error was to state that the integral of cos(0.02*t*) with respect to *t* 0.02sin(0.02*t*) rather than $\frac{\sin(0.02t)}{0.02}$. In part **(ii)** Examiners found that the majority of candidates evaluated sin(0.6) with their calculators in degree mode rather than radian mode. There were very few correct answers to part **(iii)**. Only a small number of candidates realised that the least value of *N* corresponded with the least value of sin(0.02*t*) which is –1.

Answers: **(i)** $\ln N = 50k\sin(0.02t) + \ln 125$; **(ii)** 0.0100; **(iii)** $N = 125\exp(0.502\sin(0.02t))$, 75.6.

Question 8

This question was generally well answered. The majority of marks lost in part **(a)** can be attributed to arithmetic errors. Most candidates tackled part **(i)** by multiplying the numerator and denominator by 1 + 2i. The slip of taking +8i – 3i to be –5i occurred quite frequently. Surely errors such as this could have been avoided if the work had been checked.

Most candidates attempted part **(b)** by squaring *x* + i*y*, equating the real and imaginary parts of the expansion to 5 and –12 respectively, and solving the resulting simultaneous equations. Much competent work was seen here.

Answers: **(a)(i)** 2 + I, **(ii)** $\sqrt{5}$ or 2.24, 0.464 or 26.6°; **(b)** –3 + 2i, 3 – 2i.

Question 9

Examiners reported that part **(i)** was answered confidently and well. The main difficulty in part **(ii)** seemed to be the formation of the expansion of $(2+x)^{-1}$.

Answers: **(i)** $\frac{1}{1-x} + \frac{2}{1+2x} - \frac{4}{2+x}$; **(ii)** $1 - 2x + \frac{17}{2}x^2$.

Question 10

This question differentiated well. In part **(i)**, though the equation of the plane was given in one of the standard forms, it appeared that some candidates were unfamiliar with this particular form of presentation, and consequently made errors such as taking the plane to have cartesian equation 2*x* – 3*y* + 6*z* = 0. Apart from this the work in part **(i)** was generally good. Part **(ii)** was well answered, though some candidates gave the acute angle 17.8° between the line and the normal to the plane as their final answer rather than its complement 72.2°. Part **(iii)** proved testing for many candidates. Some progress was made by those who tried to find a direction vector for the required line which was perpendicular to the line *l* and also to the normal to the plane, using simultaneous equations or a vector product. However their direction vector for *l* was not always correct, nor occasionally was their normal to the plane. Some candidates derived a correct direction vector for the required line but then gave a plane equation as their final answer.

Answers: **(i)** 3**i** + 2**j** + **k**; **(ii)** 72.2° or 1.26 radians; **(iii)** **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** – **k**).

MATHEMATICS

Paper 9709/04

Paper 4

General comments

There were fewer candidates than usual scoring extremely high marks and fewer than usual scoring extremely low marks. The main barriers to full marks were **Question 2**, **Question 3(ii)**, **Question 6(iii)**, and the effects of premature approximation.

Premature approximation was particularly prevalent in **Question 3** where θ was often taken as 30 instead of 29.7 in finding F, in **Question 5(ii)** where 0.61 was used for 0.7sin60° in finding T, in **Question 6(ii)(a)** where $(1 - 0.3)10^6$ was used instead of $(1 - \frac{1}{3})10^6$ in finding k, and in **Question 7** where 1.1 – 0.76 – 0.32 was used instead of 1.1 – 0.756 – 0.32 in finding the acceleration.

Many candidates gave answers correct to only two significant figures in **Question 1**, where 22 was often given as the answer instead of 21.9, and in **Question 3(i)** where 8.1 was given instead of 8.06.

In questions where specific symbols were defined, these were often used to represent other quantities, leading to confusion. In **Question 1** P was sometimes used to represent the driving force, in **Question 2(ii)** θ was often used to represent the angle between the resultant and some fixed direction, in **Question 5** F was often used to represent a resultant force and R to represent a resistance, which in the context of the question could only mean the frictional resistance, and in **Question 7** F was sometimes used as a resultant force, sometimes as the tension, and sometimes as the component of the weight acting down the plane.

Comments on specific questions

Question 1

This question was well attempted, although a large proportion of candidates found the rate of working to be 21 900 kW, usually represented by the statement P = 21 900 kW, without giving the required value of P as 21.9.

Answer: 21.9.

Question 2

Relatively few candidates wrote down an equation of the form $s_1(t) + s_2(t) = 10$, representing the fact that the total distance travelled by the two particles before collision is 10 m. The incorrect equation $s_1(t) = s_2(t)$, representing the idea that the particles had travelled the same distance as each other, was more common, ignoring the given 10 m. In some cases candidates wrote an incorrect equation representing the idea that the particles were moving with the same speed as each other on collision.

Answer: 6.8 m.

Question 3

(i) This part of the question was well attempted, although a significant minority of candidates failed to achieve the accuracy required by the rubric.

(ii) Candidates who demonstrated an understanding of the concepts of ‘equilibrium’ a by writing the answer directly as 7 N in the direction opposite to the now remov magnitude 7 N (or equivalent statement), were regrettably few in number. Because t answers for the magnitude and the direction are so easily accessible, it was expecte candidates who first found components *X* N and *Y* N for the resultant of the two surviving forc should then obtain the required magnitude and direction (expressed as an angle in degrees) correct to three significant figures and one decimal place respectively. This expectation was rarely achieved.

Answers: **(i)** 8.06, 29.7; **(ii)** 7 N, direction opposite to that of the force of magnitude 7 N.

Question 4

(i) Most candidates answered this part of the question correctly. However many candidates treated the problem as one of motion for a distance of 5 m in a straight line with constant acceleration 10 ms^{-2}, using $v^2 = u^2 + 2as$. Many others took the speed of *A* as zero instead of 7 ms^{-1}, obtaining the wrong answer of 10 ms^{-1}.

(ii) This part of the question was fairly well attempted, although some candidates did not consider the kinetic energy at *A* and some did not consider the change in potential energy. A significant minority of candidates confused ‘resistance’ with ‘work done against the resistance’.

Answers: **(i)** 12.2 ms^{-1}; **(ii)** 4.9 J.

Question 5

A surprisingly large number of candidates failed to realise that the frictional force acts vertically upwards. Even among those that did, many thought the normal component of the contact force also acts vertically upwards, or thought that because the normal component is horizontal the weight too must act horizontally.

(i) Candidates who realised that the frictional and normal components of the contact force act vertically and horizontally, respectively, usually produced correct expressions for *F* and *R*, although some omitted the weight and some used the weight as 4 N instead of 40 N in finding the expression for *F*. Some candidates who found an expression for *R* gave the expression for *F* as μ times the expression for *R*.

(ii) Most candidates appreciated the need to substitute their expressions for *F* and *R* into $F = 0.7R$, but of course those who omitted the weight, or who included μ in their expression for *F*, were unable to make progress. A significant number of candidates obtained an equation of the form $a + bT = cT$, but were unable to solve an equation of this type for *T*.

Answers: **(i)** $R = T\sin 60°$, $F = 40 + T\cos 60°$; **(ii)** 377.

Question 6

(i) Almost all candidates obtained correct answers and scored full marks in this part of the question.

(ii)(a) Candidates who realised the need to integrate the given $v(t)$ to obtain an expression for $s(t)$ usually did so correctly. In almost all such cases candidates continued correctly, using $s(0) = 0$ and $s(100) = 200$ to find the value of k. However a very large proportion of candidates failed to appreciate the need to use calculus, using instead formulae that apply only to motion with constant acceleration.

(b) Almost all candidates who obtained an answer in part **(ii)(a)** found the correct speed, or found a value that is correct relative to their incorrect value of *k*.

(iii) Almost all candidates sketched a correct graph for the man’s motion and very few candidates sketched a correct graph for the woman’s motion. The most usual error in the latter case was to draw a straight line, albeit starting from the origin.

Answers: **(i)** 100 s, 200 m; **(ii)(a)** 0.0003, **(b)** 3 ms^{-1}.

Question 7

(i) A surprisingly large number of candidates failed to score the mark available for this pa question. The main reasons for this failure were the omission of one or more of the four f usually the normal reaction force or the weight, and errors in the direction of one or more of forces, usually the normal reaction force being shown vertically upwards or the weight being show perpendicular to the plane.

(ii) It was expected in this part that candidates would observe from the sketch drawn in part **(i)** that, in order for P to start to move upwards, it is necessary for T to exceed the sum of F and the component of the weight of P down the plane in newtons. It is then just a matter of showing the component of weight to be 0.32 N and organising the inequality into the required form.

Unfortunately most candidates started with an equality, usually either $T = F + 0.32$ without necessarily indicating that this would be the case only if P was in equilibrium, or $T – F – 0.32 = 0.13a$. In only very few cases did candidates satisfactorily explain why the '=' sign could be replaced by > in the former case, or why this can be done in the latter case with $0.13a$ being replaced by zero.

Although the exact value of the component of weight is easily accessible from the data, many candidates obtained 0.321 N.

Many candidates thought they were required to evaluate T and F and thus to obtain a value of $T – F$ which is indeed greater than 0.32. In the most common of such answers candidates gave T as 1.1 (the weight in newtons of Q) and F as 0.32 (or 0.321) or 0.756. These values of F were obtained by ignoring the way F is defined in the question, taking F to be the component of the weight in newtons instead, or calculating the value of F assuming that μ takes the value given in part **(iii)**.

(iii) Most candidates recognised the need to apply Newton's second law to P, although fewer saw the need to apply it to Q also. The most common error in dealing with P was the omission of one term in the resultant equation, either the frictional force or the component of weight.

A very common error in dealing with Q was to assume it is in equilibrium, taking $T = 1.1$.

Answer: **(iii)** 0.1 ms^{-2}.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas. The majority of candidates had adequate time to attempt all the questions on the paper.

Most candidates worked to 3 significant figure accuracy or better and very few examples of premature approximation were seen. Nearly all candidates used the specified value of *g*.

It is pleasing to report that more candidates are now drawing their own diagrams to help them to solve the problems. Clear diagrams are helpful in tackling all questions, except **Question 4**.

Question 5 was found to be the most difficult question on the paper.

Comments on specific questions

Question 1

(i) The use of $T = \lambda \frac{x}{l}$ was very much in evidence. However, a significant number of candidates substituted the wrong value of *x*. The correct expressions were $T = 4 \times \frac{0.25}{0.25}$ or $T = 4 \times \frac{0.5}{0.5}$.

(ii) Newton's second law was applied and when used correctly quickly gave the required answer. Sometimes only one string was considered giving $T\cos\theta = 0.6a$, leading to half the value of *a*. At times the weight appeared in the equation suggesting that the strings were treated as being in a vertical plane and not on a horizontal table.

Answers: **(i)** 4 N; **(ii)** 8 ms^{-2}.

Question 2

(i) Most candidates used Newton's second law and $a = \frac{v^2}{r}$. Occasionally *T* only was seen instead of $T\sin30°$ and $r = 0.16$ was seen instead of $r = 0.16\sin30°$

(ii) This part of the question often correctly answered.

Answers: **(i)** 3.6 N; **(ii)** 0.882 N.

Question 3

(i) Most candidates attempted to take moments about *A*. Some candidates omitted the moment of the weight thereby treating the beam as a light beam. Some candidates used $\sin\alpha$ instead of $\cos\alpha$, where α was the angle with the vertical made by the string.

(ii) The principle of resolving horizontally and vertically was known and used to find *X* and *Y*. Some candidates made errors in using $\sin\alpha$ or $\cos\alpha$.

Answers: **(i)** 960 N; **(ii)** $X = 269$ and $Y = 522$.

Question 4

(i) This part of the question was generally well done. Some candidates considered it in tw firstly a retardation of $\frac{0.08v}{0.4} = 0.2v$, and then an acceleration of g due to the weight.

(ii) It was pleasing to note that most candidates knew that it was necessary to separate the variables and then to integrate. A logarithmic expression often resulted but too often the multiplier was incorrect. $v = 0$, $t = 0$ was used to find c and $t = 15$ was then substituted. Sadly the manipulation of the resulting expression to find v was often not completed successfully.

Answer: **(ii)** 47.5 ms^{-1}.

Question 5

This was a question where good diagrams would have benefited many candidates. Stronger candidates knew that they needed to consider energy. Weaker candidates tried to use tensions only.

(i) The following three-term energy equation was required:
$\frac{1}{2}\left(16 \times \frac{1^2}{0.5}\right) = \frac{1}{2} \times 0.5v + \frac{1}{2}\left(16 \times \frac{0.6^2}{0.4}\right)$.

(ii) The easiest way to solve this part of the question was to set up the equation $\frac{1}{2}\left(16 \times \frac{1^2}{0.5}\right) = \frac{1}{2}\left(\frac{16x^2}{0.4}\right)$ where x is the extension of S_1. There are alternative equations which are much more complex.

Some careless errors occurred in this question. Quite often elastic energy = $\lambda \frac{x^2}{2l}$ was quoted and x, the extension, was sometimes not squared or the 2 was omitted.

Answers: **(i)** 5.93 ms^{-1}; **(ii)** 1.29 m.

Question 6

(i) Most candidates attempted this part of the question using the trajectory equation**.** Those candidates who did not arrive at the correct velocity made errors in the manipulation of the equation. Fewer candidates tried to solve the problem by using the equations $8 = vt\cos 35°$ and $3 = vt \sin 35° - \frac{1}{2}gt$ and then eliminating t to find v.

(ii) The use of $\tan\alpha = \frac{v_y}{v_x}$ was often tried, where v_x and v_y were the horizontal and vertical components of the velocity at A and α is the angle of motion with the horizontal at A. It was pleasing to note that not many candidates used $\tan\alpha = \frac{3}{8}$.

Answers: **(i)** 13.5 ms^{-1}, 0.721; **(ii)** 2.8°or 2.9° to the horizontal.

Question 7

Part **(i)** was generally well done. Most candidates knew to take moments and those who arrived at an incorrect answer usually made an error with a wrong weight or a wrong distance.

In parts **(ii)** and **(iii)** some candidates had simply learnt that for toppling $\mu > \tan\alpha$ and for sliding $\mu < \tan\alpha$. Sometimes it was not certain that candidates had a clear understanding of the principles involved.

Answer: **(i)** 7.5.

MATHEMATICS

Paper 9709/06

Paper 6

General comments

Again there was a wide range of marks. **Question 2** was found to be easy and **Question 7** was found to be relatively difficult.

There was some evidence of premature rounding leading to a loss in accuracy marks. Some candidates did not use the last column in the normal distribution tables.

Candidates from some Centres still divided the page into two vertical sections for writing their answers – which is an undesirable practice and makes marking very difficult.

Comments on specific questions

Question 1

This first question caused difficulty with most candidates. It was one of the worst attempted on the whole paper. Many candidates clearly had not covered the work on calculating mean and standard deviations from 'given totals such as Σx and Σx^2 or $\Sigma\ (x - a)$ and $\Sigma(x - a)^2$'.

A few candidates used $n - 1$ when finding the standard deviation, which is not in the syllabus for this paper. Candidates should be aware what the words 'these values' in the question mean.

Answers: **(i)** 12; **(ii)** 8.88.

Question 2

This question was answered well by the majority of candidates. A few assumed the mean was 1, solved for p, and then calculated E(X) to be 1. They were given no marks for part **(i)** but awarded full follow-through marks for part **(ii)**. A number of candidates either forgot or did not know that the square of the mean has to be subtracted from the mean of the squares.

Answers: **(i)** $\frac{1}{6}$; **(ii)** $\frac{4}{3}$, $\frac{68}{9}$.

Question 3

A good number of candidates successfully coped with part **(i)**. There were three main ways of attempting part **(ii)** and a number of incorrect ways which fortuitously gave the correct answer. Only able candidates managed to do this part successfully.

Answers: **(i)** 120; **(ii)** 48.

Question 4

There were candidates, mainly all from particular Centres, who were unable to look probab backwards in the table to get a standardised *z*-value. These candidates thus scored no marks in pa For part **(ii)**, which was a more routine problem, candidates were awarded follow-through marks from th wrong standard deviation, thus gaining 3 marks out of 4 for this part. Many candidates wrote down the correct answer 0.595 and then used 0.6 in part **(ii)**, thus losing a mark for premature approximation. Others did not even write down 0.595 but just wrote down 0.6 thus losing a mark in part **(i)**. They then proceeded to use the 0.6 in part **(ii)** and thus lost a second mark.

Answers: **(i)** 0.595; **(ii)** 0.573.

Question 5

It was pleasing to see that the majority of candidates knew what a cumulative frequency graph was, with only a few frequency graphs appearing. Many candidates did not used the upper limits and so lost marks for the median, but were allowed marks for the interquartile range. Most candidates used sensible scales but a few went up in 51s, or 43s. It was surprising the number of candidates who did not realise that a 'time late' of –2 minutes meant that the train was early. Overall though, this question helped the weaker candidates. Some did not use their graph, as instructed in the question, to find the median and interquartile range. If the median and quartile lines were not visible on the graph then candidates lost a couple of marks. Most candidates remembered to label their axes.

Answers: **(ii)** median rounding to 2.1 – 2.4 minutes, interquartile range rounding to 3.2 – 3.6 minutes.

Question 6

Parts **(i)** and **(ii)** were usually done well. Some candidates omitted the ${}^7C_5$ and some found $P(X < 5)$ or $P(X > 5)$. This was better done than in previous years with more candidates scoring full marks. The usual confusion between mean and standard deviation and with continuity corrections were apparent, but if a candidate was well prepared they could score well in this question. Premature rounding and not using the last column in the normal distribution tables meant that many candidates lost an unnecessary mark. The last part was successfully answered by a minority of candidates.

Answers: **(i)** 0.298; **(ii)** 0.118; **(iii)** 13.

Question 7

This question was answered very well by about half the candidates, many of whom scored full marks. Surprisingly, some managed to do parts **(i)**, **(ii)** and **(iii)** successfully and then gained no marks for part **(iv)**, whilst others could not cope with the first three parts but then managed to gain full marks for part **(iv).** In general, the weaker candidates found this question demanding, with some not reading it correctly and some assuming that one paper clip was taken from box *A* and one from box *B*. They did not appear to read, or at least understand the information about transferring from one box to the other. This question proved to be a good discriminator between the weaker candidates and the stronger candidates.

Answers: **(i)** $\frac{7}{60}$; **(ii)** $\frac{47}{60}$, **(iii)** $\frac{40}{47}$; **(iv)** 0, $\frac{3}{60}$; 1, $\frac{17}{60}$; 2, $\frac{40}{60}$.

MATHEMATICS

Paper 9709/07
Paper 7

General comments

Overall, this proved to be a fairly demanding paper for weaker candidates and a good discriminator for the more able ones. There were no particular questions on this paper that were identified as being particularly easy or particularly difficult for the candidates, and candidates scored marks at varying stages throughout the paper. In the past questions on probability density functions have been well attempted by even the weakest of candidates. However, on this paper the question on this topic (**Question 5**) was not particularly straightforward and, in general, was not well attempted. **Question 2** was also not well answered by many candidates, whilst **Question 1** produced a variety of different solutions, some acceptable and some not.

There was a wide spread of marks, including some very low scores where candidates appeared to be unprepared for the examination. There were also many good scripts. Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

Candidates were able to set up their null and alternative hypotheses, and many went on to correctly find the probability of 0, 1 and 2 cars using Bin(18, 0.3), though a large proportion of these candidates did not sum the separate probabilities, or merely found the probability of 2 cars. There were also candidates who attempted to approximate to a normal distribution N(5.4, 3.78). Marks were available for this, though a continuity correction should have been applied and was not always seen. Candidates following this method did not always have a null and alternative hypothesis that corresponded to the test they were carrying out and marks were thus lost. Attempts to this question were also made using N(0.3, 0.0116). Again, successful candidates gained the marks using this method, but a mixture of different methods was all too often seen. Comparison with 0.05 (or equivalent) was not always clearly stated and was again a cause for loss of marks, as was wrong comparisons (for example *z*-values compared to areas, demonstrating a lack of understanding from some candidates). Unclear conclusions with contradictory statements were also noted by Examiners.

Answer: Accept Isaac's claim.

Question 2

Many candidates were unable to make a start to this question. Of those who attempted to standardise, many made a sign mistake and found a value for the rejection region that was greater than 3.2. Some candidates successfully found the value 2.47, but then did not state the region, or chose the wrong one. Many candidates gave the rejection region as $x < 1.645$. Part **(ii)** was poorly attempted, with many candidates not realising what was required. Errors with inequality signs were commonly noted.

Answers: **(i)** $\bar{x} < 2.47$; **(ii)** $m < 2.47$.

Question 3

Surprisingly few candidates were able to explain the meaning of the required term in part **(i)** and all t still used the word ‘random’ in their answer. However, calculating the confidence interval for the prop was better attempted, though some candidates seemed to be attempting to find a confidence interval fo mean. Use of the wrong *z*-value in the formula was occasionally seen, but on the whole the correct *z* and the correct *p* were used, though some candidates lost accuracy by using 0.37 for $\frac{130}{350}$. In part **(iii)** many candidates used an equation of the correct form and found the value of *n*, though factor of 2 errors were noted. Some candidates incorrectly formed their equation with their value of *p* as $\frac{130}{n}$ thus forming an equation in n^3. In general this question was reasonably well attempted.

Answers: **(ii)** (0.321, 0.422); **(iii)** 2241.

Question 4

Many candidates were able to find the mean correctly, but finding the variance proved problematic. Some candidates incorrectly added 800 to $5.52^2 \times 7.1^2$, but the most common error was to calculate 5.52×7.1^2. There was some confusion noted with $ and cents, whereby some candidates considering converting units unnecessarily. In part **(ii)** many candidates continued with a correct method (finding the probability of $D - 2S > 0$) but again further errors were made in finding the variance of $D - 2S$, so, while follow through marks were available for errors made in part **(i)**, some candidates were unable to score very highly on this question.

Answers: **(i)** 3360, 1540; **(ii)** 0.0693.

Question 5

The first six marks of this question were more easily accessible than the final four marks. It was only the more able candidates who realised what was required in part **(iv)**. The majority of candidates were successful in finding the mean and variance, though very weak candidates often attempted to integrate $\frac{1}{b}$ as though *b* were the variable. Non-simplified answers for both the mean and, more frequently, the variance were commonly seen. Showing that *b* was 19 was straightforward for most candidates, though candidates who had an incorrect mean often manipulated their answer in order to reach $b = 19$, and in fact would have been better merely equating their mean to 9.5 to gain the method mark. Many correct answers were seen for part **(iii)**. In part **(iv)** the more able candidates managed to find the mean and variance of the distribution of the sample means, and many went on to find the probability that the mean was less that 9 correctly. Weaker candidates merely found, as in part **(iii)**, the probability that a randomly chosen piece was less than 9.

Answers: **(i)** $\frac{b}{2}$, $\frac{b^2}{12}$; **(iii)** $\frac{8}{19}$; **(iv)** 0.0474.

Question 6

This was a reasonably well attempted question, though there was not a particular part that was consistently well done. In part **(i)** some candidates used the information given to find the correct mean of 2, though a common error was to put the factor of 3 on the wrong side of the required equation. Many candidates then went on to multiply their mean by 3.5. However, other candidates did not know how to use the given information and often used the *X*-value of either 2 or 4, or tried both, for the mean. Many candidates used the correct method to find the probability of more than 3 using their mean. Again in part **(ii)(a)** many candidates were unable to correctly use the information given to find *k*. Errors included using a new mean of $\frac{k}{1.3}$ rather than 1.3*k*, or forming an equation with P(> 1 worm) rather than P(> 0 worms). Some candidates realised that a normal distribution was required for part **(ii)(b)**, and a continuity correction was used in some cases, though incorrect ones were also noted by Examiners.

Answers: **(i)** 0.918; **(ii)(a)** 2.48, **(b)** 0.915.